

# Mathematics (MATH)

## Courses

### MATH 1000. Mathematics at Northeastern. (1 Hour)

Designed for freshman math majors to introduce them to one another, their major, their college, and the University. Students are introduced to our advising system, register for next semester's courses, and learn more about co-op. Also helps students develop the academic and interpersonal skills necessary to succeed as a university student.

### MATH 1120. Precalculus. (4 Hours)

Focuses on linear, polynomial, exponential, logarithmic, and trigonometric functions. Emphasis is placed on understanding, manipulating, and graphing these basic functions, their inverses and compositions, and using them to model real-world situations (that is, exponential growth and decay, periodic phenomena). Equations involving these functions are solved using appropriate techniques. Special consideration is given to choosing reasonable functions to fit numerical data.

### MATH 1130. College Math for Business and Economics. (4 Hours)

Introduces students to some of the important mathematical concepts and tools (such as modeling revenue, cost and profit with functions) used to solve problems in business and economics. Assumes familiarity with the basic properties of linear, polynomial, exponential, and logarithmic functions. Topics include the method of least squares, regression curves, solving equations involving functions, compound interest, amortization, and other consumer finance models. (Graphing calculator required, see instructor for make and model.)

### MATH 1213. Interactive Mathematics. (4 Hours)

Develops problem-solving skills while simultaneously teaching mathematics concepts. Each unit centers on a particular applied problem, which serves to introduce the relevant mathematical topics. These may include but are not limited to polling theory, rate of change, the concepts behind derivatives, probability, binomial distributions, and statistics. The course is not taught in the traditional lecture format and is particularly suited to students who work well in collaborative groups and who enjoy writing about the concepts they are learning. Assessment is based on portfolios, written projects, solutions to "problems of the week," and exams.

**Attribute(s):** NUpath Analyzing/Using Data, NUpath Formal/Quant Reasoning

### MATH 1215. Mathematical Thinking. (4 Hours)

Focuses on the development of mathematical thinking and its use in a variety of contexts to translate real-world problems into mathematical form and, through analysis, to obtain new information and reach conclusions about the original problems. Mathematical topics include symbolic logic, truth tables, valid arguments, counting principles, and topics in probability theory such as Bayes' theorem, the binomial distribution, and expected value.

**Attribute(s):** NUpath Analyzing/Using Data, NUpath Formal/Quant Reasoning

### MATH 1216. Recitation for MATH 1215. (0 Hours)

Provides small-group discussion format to cover material in MATH 1215.

### MATH 1220. Mathematics of Art. (4 Hours)

Presents mathematical connections and foundations for art. Topics vary and may include aspects of linear perspective and vanishing points, symmetry and patterns, tilings and polygons, Platonic solids and polyhedra, golden ratio, non-Euclidean geometry, hyperbolic geometry, fractals, and other topics. Includes connections and examples in different cultures.

**Attribute(s):** NUpath Creative Express/Innov, NUpath Formal/Quant Reasoning

### MATH 1231. Calculus for Business and Economics. (4 Hours)

Provides an overview of differential calculus including derivatives of power, exponential, logarithmic, logistic functions, and functions built from these. Derivatives are used to model rates of change, to estimate change, to optimize functions, and in marginal analysis. The integral calculus is applied to accumulation functions and future value. Emphasis is on realistic business and economics problems, the development of mathematical models from raw business data, and the translation of mathematical results into verbal expression appropriate for the business setting. Also features a semester-long marketing project in which students gather raw data, model it, and use calculus to make business decisions; each student is responsible for a ten-minute presentation. (Graphing calculator required, see instructor for make and model.)

**Attribute(s):** NUpath Formal/Quant Reasoning

### MATH 1241. Calculus 1. (4 Hours)

Serves as both the first half of a two-semester calculus sequence and as a self-contained one-semester course in differential and integral calculus. Introduces basic concepts and techniques of differentiation and integration and applies them to polynomial, exponential, log, and trigonometric functions. Emphasizes the derivative as rate of change and integral as accumulator. Applications include optimization, growth and decay, area, volume, and motion.

**Attribute(s):** NUpath Formal/Quant Reasoning

**MATH 1242. Calculus 2. (4 Hours)**

Continues MATH 1241. Introduces additional techniques of integration and numerical approximations of integrals and the use of integral tables; further applications of integrals. Also introduces differential equations and slope fields, and elementary solutions. Introduces functions of several variables, partial derivatives, and multiple integrals.

**Prerequisite(s):** MATH 1231 with a minimum grade of D- or MATH 1241 with a minimum grade of D- or MATH 1341 with a minimum grade of D-

**Attribute(s):** NUpath Formal/Quant Reasoning

**MATH 1245. Calculus with Applications. (4 Hours)**

Covers differential and integral calculus of one variable and an introduction to differential equations. Includes applications that show how calculus is used to solve problems in science. Also includes a group project related to a real-world problem in students' areas of study. The project involves a differential equation and compares the solution with experiment. Previous examples of projects include modeling of coral reefs, analysis of an epidemic using the data of the World Health Organization, analysis of a two-component kinetic model of drug concentration, and gate analysis of hip to knee experiment and comparison with the solution of the pendulum equation. Prior exposure to high-school-level calculus is recommended.

**Attribute(s):** NUpath Formal/Quant Reasoning

**MATH 1251. Calculus and Differential Equations for Biology 1. (4 Hours)**

Begins with the fundamentals of differential calculus and proceeds to the specific type of differential equation problems encountered in biological research. Presents methods for the solutions of these equations and how the exact solutions are obtained from actual laboratory data. Topics include differential calculus: basics, the derivative, the rules of differentiation, curve plotting, exponentials and logarithms, and trigonometric functions; using technology to understand derivatives; biological kinetics: zero- and first-order processes, processes tending toward equilibrium, bi- and tri-exponential processes, and biological half-life; differential equations: particular and general solutions to homogeneous and nonhomogeneous linear equations with constant coefficients, systems of two linear differential equations; compartmental problems: nonzero initial concentration, two-compartment series dilution, diffusion between compartments, population dynamics; and introduction to integration.

**Attribute(s):** NUpath Formal/Quant Reasoning

**MATH 1252. Calculus and Differential Equations for Biology 2. (4 Hours)**

Continues MATH 1251. Begins with the integral calculus and proceeds quickly to more advanced topics in differential equations. Introduces linear algebra and uses matrix methods to analyze functions of several variables and to solve larger systems of differential equations. Advanced topics in reaction kinetics are covered. The integral and differential calculus of functions of several variables is followed by the study of numerical methods in integration and solutions of differential equations. Provides a short introduction to probability. Covers Taylor polynomials and infinite series. Special topics include reaction kinetics: Michaelis-Menten processes, tracer experiments, and inflow and outflow through membranes.

**Prerequisite(s):** MATH 1251 with a minimum grade of D-

**Attribute(s):** NUpath Formal/Quant Reasoning

**MATH 1260. Math Fundamentals for Games. (4 Hours)**

Discusses linear algebra and vector geometry in two-, three-, and four-dimensional space. Examines length, dot product, and trigonometry. Introduces linear and affine transformations. Discusses complex numbers in two-space, cross product in three-space, and quaternions in four-space. Provides explicit formulas for rotations in three-space. Examines functions of one argument and treats exponentials and logarithms. Describes parametric curves in space. Discusses binomials, discrete probability, Bézier curves, and random numbers. Concludes with the concept of the derivative, the rules for computing derivatives, and the notion of a differential equation.

**Attribute(s):** NUpath Formal/Quant Reasoning

**MATH 1340. Intensive Calculus for Engineers. (6 Hours)**

Contains the material from the first semester of MATH 1341, preceded by material emphasizing the strengthening of precalculus skills. Topics include properties of exponential, logarithmic, and trigonometric functions; differential calculus; and introductory integral calculus.

**Attribute(s):** NUpath Formal/Quant Reasoning

**MATH 1341. Calculus 1 for Science and Engineering. (4 Hours)**

Covers definition, calculation, and major uses of the derivative, as well as an introduction to integration. Topics include limits; the derivative as a limit; rules for differentiation; and formulas for the derivatives of algebraic, trigonometric, and exponential/logarithmic functions. Also discusses applications of derivatives to motion, density, optimization, linear approximations, and related rates. Topics on integration include the definition of the integral as a limit of sums, antidifferentiation, the fundamental theorem of calculus, and integration by substitution.

**Attribute(s):** NUpath Formal/Quant Reasoning

**MATH 1342. Calculus 2 for Science and Engineering. (4 Hours)**

Covers further techniques and applications of integration, infinite series, and introduction to vectors. Topics include integration by parts; numerical integration; improper integrals; separable differential equations; and areas, volumes, and work as integrals. Also discusses convergence of sequences and series of numbers, power series representations and approximations, 3D coordinates, parameterizations, vectors and dot products, tangent and normal vectors, velocity, and acceleration in space. Requires prior completion of MATH 1341 or permission of head mathematics advisor.

**Attribute(s):** NUpath Formal/Quant Reasoning

**MATH 1365. Introduction to Mathematical Reasoning. (4 Hours)**

Covers the basics of mathematical reasoning and problem solving to prepare incoming math majors for more challenging mathematical courses at Northeastern. Focuses on learning to write logically sound mathematical arguments and to analyze such arguments appearing in mathematical books and courses. Includes fundamental mathematical concepts such as sets, relations, and functions.

**MATH 1990. Elective. (1-4 Hours)**

Offers elective credit for courses taken at other academic institutions. May be repeated without limit.

**MATH 2201. History of Mathematics. (4 Hours)**

Traces the development of mathematics from its earliest beginning to the present. Emphasis is on the contributions of various cultures including the Babylonians, Egyptians, Mayans, Greeks, Indians, and Arabs. Computations and constructions are worked out using the techniques and notations of these peoples. The role of mathematics in the development of science is traced throughout, including the contributions of Descartes, Kepler, Fermat, and Newton. More modern developments are discussed as time permits.

**Attribute(s):** NUpath Formal/Quant Reasoning, NUpath Interpreting Culture

**MATH 2280. Statistics and Software. (4 Hours)**

Provides an introduction to basic statistical techniques and the reasoning behind each statistical procedures. Covers appropriate statistical data analysis methods for applications in health and social sciences. Also examines a statistical package such as SPSS or SAS to implement the data analysis on computer. Topics include descriptive statistics, elementary probability theory, parameter estimation, confidence intervals, hypothesis testing, nonparametric inference, and analysis of variance and regression with a minimum of mathematical derivations.

**Attribute(s):** NUpath Analyzing/Using Data

**MATH 2321. Calculus 3 for Science and Engineering. (4 Hours)**

Extends the techniques of calculus to functions of several variables; introduces vector fields and vector calculus in two and three dimensions. Topics include lines and planes, 3D graphing, partial derivatives, the gradient, tangent planes and local linearization, optimization, multiple integrals, line and surface integrals, the divergence theorem, and theorems of Green and Stokes with applications to science and engineering and several computer lab projects. Requires prior completion of MATH 1342 or MATH 1252.

**Attribute(s):** NUpath Formal/Quant Reasoning

**MATH 2322. Recitation for MATH 2321. (0 Hours)**

Provides small-group discussion format to cover material in MATH 2321.

**MATH 2331. Linear Algebra. (4 Hours)**

Uses the Gauss-Jordan elimination algorithm to analyze and find bases for subspaces such as the image and kernel of a linear transformation. Covers the geometry of linear transformations: orthogonality, the Gram-Schmidt process, rotation matrices, and least squares fit. Examines diagonalization and similarity, and the spectral theorem and the singular value decomposition. Is primarily for math and science majors; applications are drawn from many technical fields. Computation is aided by the use of software such as Maple or MATLAB, and graphing calculators.

**Prerequisite(s):** MATH 1342 with a minimum grade of D- or MATH 1242 with a minimum grade of D- or MATH 1252 with a minimum grade of D- or CS 1800 with a minimum grade of D-

**MATH 2341. Differential Equations and Linear Algebra for Engineering. (4 Hours)**

Studies ordinary differential equations, their applications, and techniques for solving them including numerical methods (through computer labs using MS Excel and MATLAB), Laplace transforms, and linear algebra. Topics include linear and nonlinear first- and second-order equations and applications include electrical and mechanical systems, forced oscillation, and resonance. Topics from linear algebra, such as matrices, row-reduction, vector spaces, and eigenvalues/eigenvectors, are developed and applied to systems of differential equations. Requires prior completion of MATH 1342.

**MATH 2342. Recitation for MATH 2341. (0 Hours)**

Provides small-group discussion format to cover material in MATH 2341.

**MATH 2990. Elective. (1-4 Hours)**

Offers elective credit for courses taken at other academic institutions. May be repeated without limit.

**MATH 2991. Research in Mathematics. (1-4 Hours)**

Offers an opportunity to conduct introductory-level research or creative endeavors under faculty supervision.

**MATH 3000. Co-op and Experiential Learning Reflection Seminar 1. (1 Hour)**

Intended for math majors who have completed their first co-op assignment or other integrated experiential learning component of the NU Core. The goal is to examine the mathematical problems encountered in these experiences and relate them to courses already taken and to the student's future program. Faculty members and other guests contribute to the discussion. Grades are determined by the student's participation in the course and the completion of a final paper.

**MATH 3081. Probability and Statistics. (4 Hours)**

Focuses on probability theory. Topics include sample space; conditional probability and independence; discrete and continuous probability distributions for one and for several random variables; expectation; variance; special distributions including binomial, Poisson, and normal distributions; law of large numbers; and central limit theorem. Also introduces basic statistical theory including estimation of parameters, confidence intervals, and hypothesis testing.

**Prerequisite(s):** MATH 1342 with a minimum grade of D- or MATH 1252 with a minimum grade of D- or MATH 1242 with a minimum grade of D-

**Attribute(s):** NUpath Analyzing/Using Data

**MATH 3082. Recitation for MATH 3081. (0 Hours)**

Provides small-group discussion format to cover material in MATH 3081.

**MATH 3090. Exploration of Modern Mathematics. (4 Hours)**

Offers students a research-minded, elementary, and intuitive introduction to the interplay between algebra, geometry, analysis, and topology using an interactive and experimental approach. Intended for math majors, math combined majors, and students pursuing a math minor; all others should obtain permission of instructor.

**Prerequisite(s):** MATH 1242 with a minimum grade of D- or MATH 1252 with a minimum grade of D- or MATH 1342 with a minimum grade of D-

**MATH 3150. Real Analysis. (4 Hours)**

Provides the theoretical underpinnings of calculus and the advanced study of functions. Emphasis is on precise definitions and rigorous proof. Topics include the real numbers and completeness, continuity and differentiability, the Riemann integral, the fundamental theorem of calculus, inverse function and implicit function theorems, and limits and convergence. Required of all mathematics majors.

**Prerequisite(s):** (MATH 1365 with a minimum grade of D- ; MATH 2331 with a minimum grade of D- ); (ENGL 1111 with a minimum grade of C or ENGL 1102 with a minimum grade of C or ENGL 1111 with a minimum grade of C or ENGL 1102 with a minimum grade of C )

**Attribute(s):** NUpath Writing Intensive

**MATH 3175. Group Theory. (4 Hours)**

Presents basic concepts and techniques of the group theory: symmetry groups, axiomatic definition of groups, important classes of groups (abelian groups, cyclic groups, additive and multiplicative groups of residues, and permutation groups), Cayley table, subgroups, group homomorphism, cosets, the Lagrange theorem, normal subgroups, quotient groups, and direct products. Studies structural properties of groups. Possible applications include geometry, number theory, crystallography, physics, and combinatorics.

**Prerequisite(s):** MATH 2321 with a minimum grade of D- ; MATH 2331 with a minimum grade of D-

**MATH 3181. Advanced Probability and Statistics. (4 Hours)**

Focuses on probability theory needed to prepare students for research and advanced coursework in the physical and data sciences. Examples and homework problems come from physics, chemistry, biology, computer science, data science, and electrical engineering. Topics include sample spaces; conditional probability and independence; discrete and continuous probability distributions for one and for several random variables; expectation; variance; special distributions including binomial, Poisson, and normal distributions; law of large numbers; and the central limit theorem. Introduces basic statistical theory including estimation of parameters, confidence intervals, and hypothesis testing. This course is proof-based and emphasizes developing students' abilities in mathematical proof writing.

**Prerequisite(s):** MATH 2321 with a minimum grade of D-

**MATH 3275. Advanced Group Theory. (4 Hours)**

Serves as an accelerated introduction to the theory of groups, intended for students who wish to take a more advanced version of MATH 3175. Prior knowledge of group theory is not assumed. Introduces homomorphisms, subgroups, normal subgroups, quotient groups, and group actions, illustrated with a variety of examples. Subsequent topics include the class equation, simple groups, the Sylow theorems, and their applications to the classification of finite simple groups. Discusses classical matrix groups, with an emphasis on  $SU(2)$  and  $SO(3)$  as fundamental examples, and introduces the notion of a Lie algebra. Develops representation theory of finite groups and its correspondence to the representation theory of compact Lie groups sketched, again using  $SU(2)$  as an example. Students not meeting course prerequisites may seek permission of instructor.

**Prerequisite(s):** MATH 1365 with a minimum grade of B ; MATH 2331 with a minimum grade of B

**MATH 3331. Differential Geometry. (4 Hours)**

Studies differential geometry, focusing on curves and surfaces in 3D space. The material presented here can serve as preparation for a more advanced course in Riemannian geometry or differential topology.

**Prerequisite(s):** MATH 2321 with a minimum grade of D- ; MATH 2331 with a minimum grade of D-

**MATH 3341. Dynamical Systems. (4 Hours)**

Studies dynamical systems and their applications as they arise from differential equations. Solutions are obtained and analyzed as parameterized curves in the plane and used as a means of understanding the evolution of physical processes. Applications include conservative systems, predator-prey interactions, and cooperation and competition of species.

**Prerequisite(s):** MATH 2341 with a minimum grade of D-

**MATH 3527. Number Theory 1. (4 Hours)**

Introduces number theory. Topics include linear diophantine equations, congruences, design of magic squares, Fermat's little theorem, Euler's formula, Euler's phi function, computing powers and roots in modular arithmetic, the RSA encryption system, primitive roots and indices, and the law of quadratic reciprocity. As time permits, may cover diophantine approximation and Pell's equation, elliptic curves, points on elliptic curves, and Fermat's last theorem.

**Prerequisite(s):** MATH 1342 with a minimum grade of D- or MATH 1242 with a minimum grade of D- or MATH 1252 with a minimum grade of D-

**MATH 3530. Numerical Analysis. (4 Hours)**

Considers various problems including roots of nonlinear equations; simultaneous linear equations: direct and iterative methods of solution; eigenvalue problems; interpolation; and curve fitting. Emphasizes understanding issues rather than proving theorems or coming up with numerical recipes.

**Prerequisite(s):** MATH 2331 with a minimum grade of D- or MATH 2341 with a minimum grade of D-

**MATH 3533. Combinatorial Mathematics. (4 Hours)**

Introduces techniques of mathematical proofs including mathematical induction. Explores various techniques for counting such as permutation and combinations, inclusion-exclusion principle, recurrence relations, generating functions, Polya enumeration, and the mathematical formulations necessary for these techniques including elementary group theory and equivalence relations.

**Prerequisite(s):** MATH 1342 with a minimum grade of D- or MATH 1242 with a minimum grade of D- or MATH 1252 with a minimum grade of D-

**MATH 3535. Numerical Methods with Applications to Differential Equations. (4 Hours)**

Covers numerical methods to solve ordinary differential equations that are otherwise not solvable using exact methods and that are useful in a variety of applications drawn from classical areas of science and engineering. Studies examples classified as initial and boundary value problems from fluid mechanics and heat transfer. The topics are of fundamental importance in many branches of applied mathematics, physical sciences, and engineering. Possible topics also include the finite element method, which is very useful in structural analysis involving complex geometry.

**Prerequisite(s):** MATH 2341 with a minimum grade of C-

**MATH 3543. Dynamics, Chaos, and Fractals. (4 Hours)**

Introduces one-dimensional discrete real and complex dynamics, chaotic dynamics, and fractals, with an emphasis on computational and graphical exploration. Studies orbits and periodic iteration behavior of real and complex-valued function, attracting cycles, itineraries, symbolic dynamics, chaos, classical fractal constructions, Minkowski dimension, Julia sets, and the Mandelbrot set.

**Prerequisite(s):** MATH 1342 with a minimum grade of D-

**MATH 3545. Introduction to Graph Theory. (4 Hours)**

Offers a mathematical introduction to networks and graphs, which find applications in social and natural sciences. Introduces paths, cycles, trees, bipartite graphs, matchings, colorings, connectivity, and network flows. Discusses special cases of planar, Eulerian, and Hamiltonian graphs; Tait's theorem; and possible advanced topics. Students who do not meet course prerequisites may seek permission of instructor.

**Prerequisite(s):** MATH 1365 with a minimum grade of C- or MATH 2310 with a minimum grade of C- or MATH 3533 with a minimum grade of C- or CS 1800 with a minimum grade of C- or CS 3800 with a minimum grade of C-

**MATH 3560. Geometry. (4 Hours)**

Studies classical geometry and symmetry groups of geometric figures, with an emphasis on Euclidean geometry. Teaches how to formulate mathematical propositions precisely and how to construct and understand mathematical proofs. Provides a line between classical and modern geometry with the aim of preparing students for further study in group theory and differential geometry.

**Prerequisite(s):** MATH 2331 with a minimum grade of D- or MATH 2341 with a minimum grade of D-

**MATH 3990. Elective. (1-4 Hours)**

Offers elective credit for courses taken at other academic institutions. May be repeated without limit.

**MATH 4020. Research Capstone. (4 Hours)**

Offers students the experience of engaging in mathematical research that builds upon the math courses that they have taken and, possibly, their co-op assignments. Requires students to complete a research project of their own choosing. Focus is on the project and on the students presenting their work. Also requires students to write a reflection paper. Intended for juniors or seniors with experience or interest in mathematics research. Students who do not meet course prerequisites may seek permission of instructor.

**Prerequisite(s):** MATH 3150 with a minimum grade of D- ; MATH 3175 with a minimum grade of D-

**Attribute(s):** NUpath Capstone Experience, NUpath Writing Intensive

**MATH 4025. Applied Mathematics Capstone. (4 Hours)**

Emphasizes the use of a variety of methods—such as optimization, differential equations, probability, and statistics—to study problems that arise in epidemiology, finance, and other real-world settings. Course work includes assigned exercises, a long-term modeling project on a topic of the student's choosing, and a reflection paper.

**Prerequisite(s):** MATH 3081 with a minimum grade of D-

**Attribute(s):** NUpath Capstone Experience, NUpath Writing Intensive

**MATH 4525. Applied Analysis. (4 Hours)**

Demonstrates the applications of mathematics to interesting physical and biological problems. Methods are chosen from ordinary and partial differential equations, calculus of variations, Laplace transform, perturbation theory, special functions, dimensional analysis, asymptotic analysis, and other techniques of applied mathematics.

**Prerequisite(s):** MATH 2321 with a minimum grade of D- ; MATH 2331 with a minimum grade of D- ; (MATH 2341 with a minimum grade of D- or MATH 2351 with a minimum grade of D- )

**MATH 4527. Number Theory 2. (4 Hours)**

Continues MATH 3527. Topics include Diophantine approximation, the Gaussian integers, irrational numbers and transcendental numbers, nonlinear polynomial congruences, systems of linear congruences, mobius inversion, elliptic curves, modular curves, modular forms, and L-functions.

**Prerequisite(s):** (MATH 3527 with a minimum grade of D- or MATH 4575 with a minimum grade of D- ); MATH 3175 with a minimum grade of D-

**MATH 4541. Advanced Calculus. (4 Hours)**

Offers a deeper and more generalized look at the ideas and objects of study of calculus. Topics include the generalized calculus of n-space, the inverse and implicit function theorems, differential forms and general Stokes-type theorems, geometry of curves and surfaces, and special functions.

**Prerequisite(s):** MATH 2321 with a minimum grade of D- ; MATH 2331 with a minimum grade of D-

**MATH 4545. Fourier Series and PDEs. (4 Hours)**

Provides a first course in Fourier series, Sturm-Liouville boundary value problems, and their application to solving the fundamental partial differential equations of mathematical physics: the heat equation, the wave equation, and Laplace's equation. Green's functions are also introduced as a means of obtaining closed-form solutions.

**Prerequisite(s):** MATH 2351 with a minimum grade of D- or MATH 2341 with a minimum grade of D-

**MATH 4555. Complex Variables. (4 Hours)**

Provides an introduction to the analysis of functions of a complex variable. Starting with the algebra and geometry of complex numbers, basic derivative and contour integral properties are developed for elementary algebraic and transcendental functions as well as for other analytic functions and functions with isolated singularities. Power and Laurent series representations are given. Classical integral theorems, residue theory, and conformal mapping properties are studied. Applications of harmonic functions are presented as time permits.

**Prerequisite(s):** MATH 2321 with a minimum grade of D-

**MATH 4565. Topology. (4 Hours)**

Introduces the student to fundamental notions of topology. Introduces basic set theory, then covers the foundations of general topology (axioms for a topological space, continuous functions, homeomorphisms, metric spaces, the subspace, product and quotient topologies, connectedness, compactness, and the Hausdorff condition). Also introduces algebraic and geometric topology (homotopy, covering spaces, fundamental groups, graphs, surfaces, and manifolds) and applications. Other topics are covered if time permits.

**Prerequisite(s):** MATH 3150 with a minimum grade of D-

**MATH 4570. Matrix Methods in Data Analysis and Machine Learning. (4 Hours)**

Introduces concepts and methods of linear algebra for understanding and creating machine learning and deep learning algorithms. Topics include various matrix factorizations, symmetric positive definite matrices, inner product spaces, matrix calculus, applications to probability and statistics, and optimization in high-dimensional spaces. Explores the mathematics behind data analysis, machine learning, and deep learning, including gradient descents, Newton's methods, principal components analysis, linear regression and linear methods in classification, neural networks, and convolutional neural networks. Offers students opportunities to learn and practice Python skills with labs and the final project.

**Prerequisite(s):** MATH 2331 with a minimum grade of C- or MATH 2341 with a minimum grade of C-

**MATH 4571. Advanced Linear Algebra. (4 Hours)**

Provides a more detailed study of linear transformations and matrices: LU factorization, QR factorization, Spectral theorem and singular value decomposition, Jordan form, positive definite matrices, quadratic forms, partitioned matrices, and norms and numerical issues. Topics and emphasis change from year to year.

**Prerequisite(s):** MATH 2331 with a minimum grade of D-

**MATH 4575. Introduction to Cryptography. (4 Hours)**

Presents the mathematical foundations of cryptology, beginning with the study of divisibility of integers, the Euclidian Algorithm, and an analysis of the Extended Euclidian Algorithm. Includes a short study of groups, semigroups, residue class rings, fields, Fermat's Little Theorem, Chinese Remainder Theorem, polynomials over fields, and the multiplicative group of residues modulo a prime number. Introduces fundamental notions used to describe encryption schemes together with examples, which include affine linear ciphers and cryptanalysis and continues with probability and perfect secrecy. Presents the Data Encryption Standard (DES) and culminates in the study of the Advanced Encryption Standard (AES), the standard encryption scheme in the United States since 2001.

**Prerequisite(s):** MATH 2331 with a minimum grade of D- or MATH 3175 with a minimum grade of D- or MATH 3527 with a minimum grade of D-

**MATH 4576. Rings and Fields. (4 Hours)**

Introduces commutative rings, ideals, integral domains, fields, and the theory of extension fields. Topics include Gaussian integers, Galois groups, and the fundamental theorem of Galois theory. Applications include the impossibility of angle-trisection and the general insolubility of fifth- and higher-degree polynomials. Other topics are covered as time permits.

**Prerequisite(s):** MATH 3175 with a minimum grade of D-

**MATH 4577. Commutative Algebra. (4 Hours)**

Introduces the basics of commutative algebra. Emphasizes rigorously building the mathematical background needed for studying this subject in more depth. Seeks to prepare students for more advanced classes in algebraic geometry, robotics, invariant theory of finite groups, and cryptography. Covers geometry, algebra, and algorithms; Grobner bases; elimination theory; the algebra-geometry dictionary; robotics and automatic geometric theorem proving.

**Prerequisite(s):** (MATH 2331 with a minimum grade of C+ ; MATH 1365 with a minimum grade of C+ ) or MATH 3175 with a minimum grade of C+ or MATH 4576 (may be taken concurrently) with a minimum grade of C+

**MATH 4581. Statistics and Stochastic Processes. (4 Hours)**

Continues topics introduced in MATH 3081. The first part of the course covers classical procedures of statistics including the t-test, linear regression, and the chi-square test. The second part provides an introduction to stochastic processes with emphasis on Markov chains, random walks, and Brownian motion, with applications to modeling and finance.

**Prerequisite(s):** MATH 3081 with a minimum grade of D-

**MATH 4606. Mathematical and Computational Methods for Physics. (4 Hours)**

Covers advanced mathematical methods topics that are commonly used in the physical sciences, such as complex calculus, Fourier transforms, special functions, and the principles of variational calculus. Applies these methods to computational simulation and modeling exercises. Introduces basic computational techniques and numerical analysis, such as Newton's method, Monte Carlo integration, gradient descent, and least squares regression. Uses a simple programming language, such as MATLAB, for the exercises.

**Prerequisite(s):** PHYS 2303 with a minimum grade of D- ; MATH 2321 with a minimum grade of D- ; (MATH 2341 with a minimum grade of D- or MATH 2351 with a minimum grade of D- )

**MATH 4681. Probability and Risks. (4 Hours)**

Reviews main probability and statistics concepts from the point of view of decision risks in actuarial and biomedical contexts, including applications of normal approximation for evaluating statistical risks. Also examines new topics, such as distribution of extreme values and nonparametric statistics with examples. May be especially useful for students preparing for the first actuarial exam on probability and statistics.

**Prerequisite(s):** MATH 3081 with a minimum grade of D-

**MATH 4682. Theory of Interest and Basics of Life Insurance 1. (4 Hours)**

Reviews basic financial instruments in the presence of interest rates, including the measurement of interest and problems in interest (equations of value, basic and more general annuities, yield rates, amortization schedules, bonds and other securities). Examines numerous practical applications. Also introduces problems of life insurance with examples. May be especially useful for students preparing for the second actuarial exam on theory of interest.

**Prerequisite(s):** MATH 3081 with a minimum grade of D-

**MATH 4683. Financial Derivatives. (4 Hours)**

Presents the mathematical basis of actuarial models and their application to insurance and other financial risks. Includes but is not limited to financial derivatives such as options and futures. Techniques and applications may be useful for students preparing for actuarial Exam 3F (Society of Actuaries Exam MFE).

**Prerequisite(s):** MATH 4581 with a minimum grade of D-

**MATH 4684. Theory of Interest and Basics of Life Insurance 2. (4 Hours)**

Reviews actuarial models for life insurance, including survival models, life tables, life insurance, life annuities, premium and policy values, multiple state models, joint-life and last survivor models, pensions, and emerging costs for life insurance. Designed to be especially useful for students preparing for the fourth actuarial exam, LTAM (Long-Term Actuarial Mathematics), of the Society of Actuaries.

**Prerequisite(s):** MATH 4682 with a minimum grade of C-

**MATH 4970. Junior/Senior Honors Project 1. (4 Hours)**

Focuses on in-depth project in which a student conducts research or produces a product related to the student's major field. Combined with Junior/Senior Project 2 or college-defined equivalent for 8-credit honors project. May be repeated without limit.

**Prerequisite(s):** MATH 3081 with a minimum grade of D-

**MATH 4971. Junior/Senior Honors Project 2. (4 Hours)**

Focuses on second semester of in-depth project in which a student conducts research or produces a product related to the student's major field. May be repeated without limit.

**Prerequisite(s):** MATH 4970 with a minimum grade of D-

**MATH 4990. Elective. (1-4 Hours)**

Offers elective credit for courses taken at other academic institutions. May be repeated without limit.

**MATH 4991. Research. (4 Hours)**

Offers an opportunity to conduct research under faculty supervision.

**Attribute(s):** NUpath Integration Experience

**MATH 4992. Directed Study. (1-4 Hours)**

Offers independent work under the direction of members of the department on a chosen topic. Course content depends on instructor. May be repeated without limit.

**MATH 4994. Internship. (4 Hours)**

Offers students an opportunity for internship work. May be repeated without limit.

**Attribute(s):** NUpath Integration Experience

**MATH 4996. Experiential Education Directed Study. (4 Hours)**

Draws upon the student's approved experiential activity and integrates it with study in the academic major. Restricted to mathematics majors who are using it to fulfill their experiential education requirement; for these students it may count as a mathematics elective, subject to approval by instructor and adviser. May be repeated without limit.

**Attribute(s):** NUpath Integration Experience, NUpath Writing Intensive

**MATH 5101. Analysis 1: Functions of One Variable. (4 Hours)**

Offers a rigorous, proof-based introduction to mathematical analysis and its applications. Topics include metric spaces, convergence, compactness, and connectedness; continuous and uniformly continuous functions; derivatives, the mean value theorem, and Taylor series; Riemann integration and the fundamental theorem of calculus; interchanging limit operations; sequences of functions and uniform convergence; Arzelà-Ascoli and Stone-Weierstrass theorems; inverse and implicit function theorems; successive approximations and existence/uniqueness for ordinary differential equations; linear operators on finite-dimensional vector spaces and applications to systems of ordinary differential equations. Provides a series of computer projects that further develop the connections between theory and applications. Requires permission of instructor and head advisor for undergraduate students.

**MATH 5102. Analysis 2: Functions of Several Variables. (4 Hours)**

Continues MATH 5101. Studies basics of analysis in several variables. Topics include derivative and partial derivatives; the contraction principle; the inverse function and implicit function theorems; derivatives of higher order; Taylor formula in several variables; differentiation of integrals depending on parameters; integration of functions of several variables; change of variables in integrals; differential forms and their integration over simplexes and chains; external multiplication of forms; differential of forms; Stokes' formula; set functions; Lebesgue measure; measure spaces; measurable functions; integration; comparison with the Riemann integral;  $L_2$  as a Hilbert space; and Parseval theorem and Riesz-Fischer theorem. Requires permission of instructor and head advisor for undergraduate students.

**MATH 5110. Applied Linear Algebra and Matrix Analysis. (4 Hours)**

Offers a robust introduction to the basic results of linear algebra on real and complex vector spaces with applications to differential equations and Markov chains. Introduces theoretical results along the way, along with matrix analysis, eigenvalue analysis, and spectral decomposition. Includes a significant computational component, focused on applications of linear algebra to mathematical modeling.

**MATH 5111. Algebra 1. (4 Hours)**

Discusses fundamentals of the theory of groups and some applications in Galois theory. Topics may include quotient groups and isomorphism theorems; group actions and Sylow theory; simplicity and solvability; permutations and the simplicity of the higher alternating groups; fields and polynomial rings; splitting fields; the Galois correspondence; computations of Galois groups; and applications of Galois theory, including non-solvability of polynomials by radicals.

**MATH 5112. Algebra 2. (4 Hours)**

Provides a comprehensive introduction to commutative algebra: rings and modules. Topics in commutative ring theory include ideals, prime and maximal ideals, ring homomorphisms, Euclidean domains, principal ideal domains, unique factorization domains, fields of quotients, polynomial rings, irreducibility criteria, and the Chinese Remainder Theorem. Topics in module theory include module homomorphisms, the structure theorem for modules over a PID and applications, and the Jordan and rational canonical forms.

**MATH 5121. Topology 1. (4 Hours)**

Provides an introduction to topology, starting with the basics of point set topology (topological space, continuous maps, homeomorphisms, compactness and connectedness, and identification spaces). Moves on to the basic notions of algebraic and combinatorial topology, such as homotopy equivalences, fundamental group, Seifert-VanKampen theorem, simplicial complexes, classification of surfaces, and covering space theory. Ends with a brief introduction to simplicial homology and knot theory. Requires permission of instructor and head advisor for undergraduate students.

**Prerequisite(s):** MATH 5111 with a minimum grade of C-

**MATH 5122. Geometry 1. (4 Hours)**

Covers differentiable manifolds, such as tangent bundles, tensor bundles, vector fields, Frobenius integrability theorem, differential forms, Stokes' theorem, and de Rham cohomology; and curves and surfaces, such as elementary theory of curves and surfaces in  $\mathbb{R}^3$ , fundamental theorem of surfaces in  $\mathbb{R}^3$ , surfaces with constant Gauss or mean curvature, and Gauss-Bonnet theorem for surfaces. Requires permission of instructor and head advisor for undergraduate students.

**Prerequisite(s):** MATH 5101 with a minimum grade of C- ; MATH 5111 with a minimum grade of C-

**MATH 5131. Introduction to Mathematical Methods and Modeling. (4 Hours)**

Presents mathematical methods emphasizing applications. Uses ordinary and partial differential equations to model the evolution of real-world processes. Topics chosen illustrate the power and versatility of mathematical methods in a variety of applied fields and include population dynamics, drug assimilation, epidemics, spread of pollutants in environmental systems, competing and cooperating species, and heat conduction. Requires students to complete a math-modeling project. Requires undergraduate-level course work in ordinary and partial differential equations.

**Attribute(s):** NUpath Capstone Experience, NUpath Writing Intensive

**MATH 5352. Quantum Computation and Information. (4 Hours)**

Introduces the foundations of quantum computation and information, including finite dimensional quantum mechanics, gates and circuits, quantum algorithms, quantum noise, and error-correcting codes. Assumes a working knowledge of linear algebra and matrix analysis, but no prior experience with quantum theory or algorithms is required.

**MATH 6000. Professional Development for Co-op. (0 Hours)**

Introduces the cooperative education program. Offers students an opportunity to develop job-search and career-management skills; to assess their workplace skills, interests, and values and to discuss how they impact personal career choices; to prepare a professional resumé; and to learn proper interviewing techniques. Explores career paths, choices, professional behaviors, work culture, and career decision making.

**MATH 6954. Co-op Work Experience - Half-Time. (0 Hours)**

Provides eligible students with an opportunity for work experience. May be repeated without limit.

**MATH 6955. Co-op Work Experience Abroad - Half Time. (0 Hours)**

Provides eligible students with an opportunity for work experience abroad.

**MATH 6961. Internship. (1-4 Hours)**

Offers students an opportunity for internship work. May be repeated without limit.

**MATH 6962. Elective. (1-4 Hours)**

Offers elective credit for courses taken at other academic institutions. May be repeated without limit.

**MATH 6964. Co-op Work Experience. (0 Hours)**

Provides eligible students with an opportunity for work experience. May be repeated without limit.

**MATH 6965. Co-op Work Experience Abroad. (0 Hours)**

Provides eligible students with an opportunity for work experience abroad. May be repeated without limit.

**MATH 7202. Partial Differential Equations 1. (4 Hours)**

Introduces partial differential equations, their theoretical foundations, and their applications, which include optics, propagation of waves (light, sound, and water), electric field theory, and diffusion. Topics include first-order equations by the method of characteristics; linear, quasilinear, and nonlinear equations; applications to traffic flow and geometrical optics; principles for higher-order equations; power series and Cauchy-Kowalevski theorem; classification of second-order equations; linear equations and generalized solutions; wave equations in various space dimensions; domain of dependence and range of influence; Huygens' principle; conservation of energy, dispersion, and dissipation; Laplace's equation; mean values and the maximum principle; the fundamental solution, Green's functions, and Poisson kernels; applications to physics; properties of harmonic functions; the heat equation; eigenfunction expansions; the maximum principle; Fourier transform and the Gaussian kernel; regularity of solutions; scale invariance and the similarity method; Sobolev spaces; and elliptic regularity.

**MATH 7203. Numerical Analysis 1. (4 Hours)**

Introduces methods and techniques used in contemporary number crunching. Covers floating-point computations involving scalars, vectors, and matrices; solvers for sparse and dense linear systems; matrix decompositions; integration of functions and solutions of ordinary differential equations (ODEs); and Fast Fourier transform. Focuses on finding solutions to practical, real-world problems. Knowledge of programming in Matlab is assumed. Knowledge of other programming languages would be good but not required.

**MATH 7205. Numerical Analysis 2. (4 Hours)**

Covers numerical analysis and scientific computation. Topics include numerical solutions of ordinary differential equations (ODEs) and one-dimensional boundary value problems; solving partial differential equations (PDEs) using modal expansions, finite-difference, and finite-element methods; stability of PDE algorithms; elementary computational geometry and mesh generation; unconstrained optimization with application to data modeling; and constrained optimization of convex functions: linear and quadratic programming. Focuses on techniques commonly used for data fitting and solving problems from engineering and physical science. Knowledge of programming in MATLAB is assumed. Knowledge of other programming languages beneficial but not required.

**MATH 7221. Topology 2. (4 Hours)**

Continues MATH 5121. Introduces homology and cohomology theory. Studies singular homology, homological algebra (exact sequences, axioms), Mayer-Vietoris sequence, CW-complexes and cellular homology, calculation of homology of cellular spaces, and homology with coefficients. Moves on to cohomology theory, universal coefficients theorems, Bockstein homomorphism, Kunneth formula, cup and cap products, Hopf invariant, Borsuk-Ulam theorem, and Brouwer and Lefschetz-Hopf fixed-point theorems. Ends with a study of duality in manifolds including orientation bundle, Poincaré duality, Lefschetz duality, Alexander duality, Euler class, Lefschetz numbers, Gysin sequence, intersection form, and signature.

**MATH 7223. Riemannian Optimization. (4 Hours)**

Offers a self-contained introduction to optimization on smooth manifolds. Covers both theoretical foundations and practical computational methods that students can apply to their own work. Introduces the theory of Riemannian geometry. Emphasizes those elements that are relevant for the construction of optimization algorithms (tensor fields, metrics, connections, geodesics, retractions, and transporters). Applies this geometric machinery to devise first- and second-order smooth optimization methods on generic Riemannian manifolds. Focuses on the development of practical computational techniques, with applications to robotics, computer vision, and machine learning.

**MATH 7233. Graph Theory. (4 Hours)**

Covers fundamental concepts in graph theory. Topics include adjacency and incidence matrices, paths and connectedness, and vertex degrees and counting; trees and distance including properties of trees, distance in graphs, spanning trees, minimum spanning trees, and shortest paths; matchings and factors including matchings in bipartite graphs, Hall's matching condition, and min-max theorems; connectivity, such as vertex connectivity, edge connectivity,  $k$ -connected graphs, and Menger's theorem; network flows including maximum network flow, and integral flows; vertex colorings, such as upper bounds, Brooks' theorem, graphs with large chromatic number, and critical graphs; Eulerian circuits and Hamiltonian cycles including Euler's theorem, necessary conditions for Hamiltonian cycles, and sufficient conditions; planar graphs including embeddings and Euler's formula, characterization of planar graphs (Kuratowski's theorem); and Ramsey theory including Ramsey's theorem, Ramsey numbers, and graph Ramsey theory.

**MATH 7234. Optimization and Complexity. (4 Hours)**

Offers theory and methods of maximizing and minimizing solutions to various types of problems. Studies combinatorial problems including mixed integer programming problems (MIP); pure integer programming problems (IP); Boolean programming problems; and linear programming problems (LP). Topics include convex subsets and polyhedral subsets of  $n$ -space; relationship between an LP problem and its dual LP problem, and the duality theorem; simplex algorithm, and Kuhn-Tucker conditions for optimality for nonlinear functions; and network problems, such as minimum cost and maximum flow-minimum cut. Also may cover complexity of algorithms; problem classes P (problems with polynomial-time algorithms) and NP (problems with nondeterministic polynomial-time algorithms); Turing machines; and NP-completeness of traveling salesman problem and other well-known problems.

**MATH 7241. Probability 1. (4 Hours)**

Offers an introductory course in probability theory, with an emphasis on problem solving and modeling. Starts with basic concepts of probability spaces and random variables, and moves on to the classification of Markov chains with applications. Other topics include the law of large numbers and the central limit theorem, with applications to the theory of random walks and Brownian motion.

**MATH 7243. Machine Learning and Statistical Learning Theory 1. (4 Hours)**

Introduces both the mathematical theory of learning and the implementation of modern machine-learning algorithms appropriate for data science. Modeling everything from social organization to financial predictions, machine-learning algorithms allow us to discover information about complex systems, even when the underlying probability distributions are unknown. Algorithms discussed include regression, decision trees, clustering, and dimensionality reduction. Offers students an opportunity to learn the implications of the mathematical choices underpinning the use of each algorithm, how the results can be interpreted in actionable ways, and how to apply their knowledge through the analysis of a variety of data sets and models.

**MATH 7301. Functional Analysis. (4 Hours)**

Provides an introduction to essential results of functional analysis and some of its applications. The main abstract facts can be understood independently. Proof of some important basic theorems about Hilbert and Banach spaces (Hahn-Banach theorem, open mapping theorem) are omitted, in order to allow more time for applications of the abstract techniques, such as compact operators; Peter-Weyl theorem for compact groups; spectral theory; Gelfand's theory of commutative  $C^*$ -algebras; mean ergodic theorem; Fourier transforms and Sobolev embedding theorems; and distributions and elliptic operators.

**Prerequisite(s):** MATH 5102 with a minimum grade of C- or MATH 5102 with a minimum grade of C-

**MATH 7302. Partial Differential Equations 2. (4 Hours)**

Covers advanced topics in linear and nonlinear partial differential equations. Topics include pseudodifferential operators and elliptic regularity; elements of microlocal analysis; propagation of singularities; spectral theory of elliptic operators; variational principle; the Schrödinger equation and its meaning in quantum mechanics; parabolic equations and their role in diffusion processes; hyperbolic equations and wave propagation; the Cauchy problem for hyperbolic equations; elements of scattering theory; nonlinear elliptic equations in Riemannian geometry, including the Yamabe problem, prescribed scalar curvature problem, and Einstein-Kähler metrics; the Navier-Stokes equations in hydrodynamics; simplest properties and open problems in nonlinear hyperbolic equations and shock waves; the Korteweg-de Vries equation and its relation to inverse scattering problems; solitons and algebro-geometric solutions.

**Prerequisite(s):** MATH 5101 with a minimum grade of C-

**MATH 7303. Complex Manifolds. (4 Hours)**

Introduces complex manifolds. Discusses the elementary local theory in several variables including Cauchy's integral formula, Hartog's extension theorem, the Weierstrass preparation theorem, and Riemann's extension theorem. The global theory includes the definition of complex manifolds, sheaf cohomology, line bundles and divisors, Kodaira's vanishing theorem, Kodaira's embedding theorem, and Chow's theorem on complex subvarieties of projective space. Special examples of dimension one and two illustrate the general theory.

**MATH 7311. Commutative Algebra. (4 Hours)**

Introduces some of the main tools of commutative algebra, particularly those tools related to algebraic geometry. Topics include prime ideals, localization, and integral extensions; primary decomposition; Krull dimension; chain conditions, and Noetherian and Artinian modules; and additional topics from ring and module theory as time permits.

**Prerequisite(s):** MATH 5111 with a minimum grade of C-

**MATH 7315. Algebraic Number Theory. (4 Hours)**

Covers rings of integers, Dedekind domains, factorization of ideals, ramification, and the decomposition and inertia subgroups; units in rings of integers, Minkowski's geometry of numbers, and Dirichlet's unit theorem; and class groups, zeta functions, and density sets of primes.

**Prerequisite(s):** MATH 5111 with a minimum grade of C-

**MATH 7316. Lie Algebras. (4 Hours)**

Introduces notions of solvable and nilpotent Lie algebras. Covers semisimple Lie algebras: Killing form criterion, Cartan decomposition, root systems, Weyl groups, Dynkin diagrams, weights. Also discusses universal enveloping algebra, PBW theorem, representations of semisimple Lie algebras, weight spaces, highest weight modules, multiplicities, characters, Weyl character formula.

**MATH 7317. Modern Representation Theory. (4 Hours)**

Introduces students to modern techniques of representation theory, including those coming from geometry and mathematical physics. Covers applications of geometry to the representation theory of semisimple Lie algebras, algebraic groups and related algebraic objects, questions related to the representation theory of infinite dimensional Lie algebras, quantum groups, and  $p$ -adic groups, as well as category theory methods in representation theory.

**Prerequisite(s):** MATH 5111 with a minimum grade of C-

**MATH 7320. Modern Algebraic Geometry. (4 Hours)**

Introduces students to modern techniques of algebraic geometry, including those coming from Lie theory, symplectic and differential geometry, complex analysis, and number theory. Covers subjects related to invariant theory, homological algebra questions of algebraic geometry, including derived categories and complex analytic, differential geometric, and arithmetic aspects of the geometry of algebraic varieties. Students not meeting course prerequisites or restrictions may seek permission of instructor.

**Prerequisite(s):** MATH 7314 with a minimum grade of C- or MATH 7361 with a minimum grade of C-

**MATH 7321. Topology 3. (4 Hours)**

Continues MATH 7221 and studies classical algebraic topology and its applications. Introduces homotopy theory. Topics include higher homotopy groups, cofibrations, fibrations, homotopy sequences, homotopy groups of Lie groups and homogeneous spaces, Hurewicz theorem, Whitehead theorem, Eilenberg-MacLane spaces, obstruction theory, Postnikov towers, and spectral sequences.

**Prerequisite(s):** MATH 7221 with a minimum grade of C-

**MATH 7339. Machine Learning and Statistical Learning Theory 2. (4 Hours)**

Continues MATH 7243. Further covers theory and methods for regression and classification, along with more advanced topics in machine learning, statistical learning, and deep learning. Reviews the basics of machine learning in a broader and deeper way. Additional topics are drawn from smoothing methods, clustering, latent variable models, mixture models, Markov decision process and reinforcement learning, and neural networks. Discusses recent research papers on image classification and segmentation, generative adversarial network, neural style transfer, natural language processing, and topological data analysis. Uses theory, models, and algorithms to analyze a variety of datasets.

**Prerequisite(s):** CS 6140 with a minimum grade of C- or DS 5220 with a minimum grade of C- or EECE 5644 with a minimum grade of C- or MATH 7243 with a minimum grade of C-

**MATH 7340. Statistics for Bioinformatics. (4 Hours)**

Introduces the concepts of probability and statistics used in bioinformatics applications, particularly the analysis of microarray data. Uses statistical computation using the open-source R program. Topics include maximum likelihood; Monte Carlo simulations; false discovery rate adjustment; nonparametric methods, including bootstrap and permutation tests; correlation, regression, ANOVA, and generalized linear models; preprocessing of microarray data and gene filtering; visualization of multivariate data; and machine-learning techniques, such as clustering, principal components analysis, support vector machine, neural networks, and regression tree.

**MATH 7341. Probability 2. (4 Hours)**

Continues MATH 7241. Studies probability theory, with an emphasis on its use in modeling and queueing theory. Starts with basic properties of exponential random variables, and then applies this to the study of the Poisson process. Queueing theory forms the bulk of the course, with analysis of single-server queues, multiserver queues, and networks of queues. Also includes material on continuous-time Markov processes, renewal theory, and Brownian motion.

**Prerequisite(s):** MATH 7241 with a minimum grade of C- or IE 6200 with a minimum grade of C-

**MATH 7342. Mathematical Statistics. (4 Hours)**

Introduces mathematical statistics, emphasizing theory of point estimations. Topics include parametric estimations, minimum variance unbiased estimators, sufficiency and completeness, and Rao-Blackwell theorem; asymptotic (large sample) theory, maximum likelihood estimator (MLE), consistency of MLE, asymptotic theory of MLE, and Cramer-Rao bound; and hypothesis testing, Neyman-Pearson fundamental lemma, and likelihood ratio test.

**MATH 7343. Applied Statistics. (4 Hours)**

Designed as a basic introductory course in statistical methods for graduate students in mathematics as well as various applied sciences. Topics include descriptive statistics, inference for population means, analysis of variance, nonparametric methods, and linear regression. Studies how to use the computer package SPSS, doing statistical analysis and interpreting computer outputs.

**MATH 7344. Regression, ANOVA, and Design. (4 Hours)**

Discusses one-sample and two-sample tests; one-way ANOVA; factorial and nested designs; Cochran's theorem; linear and nonlinear regression analysis and corresponding experimental design; analysis of covariance; and simultaneous confidence intervals.

**MATH 7349. Stochastic Calculus and Introduction to No-Arbitrage Finance. (4 Hours)**

Introduces no-arbitrage discounted contingent claims and methods of their optimization in discrete and continuous time for a finite fixed or random horizon. Establishes the relation of no-arbitrage to the martingale calculus. Introduces stochastic differential equations and corresponding PDE describing functionals of their solutions. Presents examples of contingent claims (such as options) evaluation including the Black-Scholes formula.

**Prerequisite(s):** MATH 7342 with a minimum grade of C- or MATH 7343 with a minimum grade of C-

**MATH 7351. Mathematical Methods of Classical Mechanics. (4 Hours)**

Overviews the mathematical formulation of classical mechanics. Topics include Hamilton's principle and Lagrange's equations; solution of the two-body central force problem; rigid body rotation and Euler's equations; the spinning top; Hamilton's equations; the Poisson bracket; Liouville's theorem; and canonical transformations.

**MATH 7352. Mathematical Methods of Quantum Mechanics. (4 Hours)**

Introduces the basics of quantum mechanics for mathematicians. Introduces the von Neumann's axiomatics of quantum mechanics with measurements in the first part of the course. Discusses the notions of observables and states, as well as the connections between the quantum and the classical mechanics. The second (larger) part is dedicated to some concrete quantum mechanical problems, such as harmonic oscillator, one-dimensional problems of quantum mechanics, radial Schrödinger equation, and the hydrogen atom. The third part deals with more advanced topics, such as perturbation theory, scattering theory, and spin. Knowledge of functional analysis and classical mechanics recommended.

**Prerequisite(s):** (MATH 5102 with a minimum grade of C- or MATH 5102 with a minimum grade of D-); (MATH 5111 with a minimum grade of C- or MATH 5111 with a minimum grade of D-)

**MATH 7359. Elliptic Curves and Modular Forms. (4 Hours)**

Introduces elliptic curves and modular forms. Examines elliptic curves as algebraic varieties over the complex numbers, finite fields, and local and global fields. Related topics include the  $j$ -invariant, the Tate module and the Weil pairing, zeta functions and the Weil conjectures, and the Mordell-Weil theorem. Modular forms are defined on moduli spaces of elliptic curves. Related topics include Eisenstein series, cusp forms, congruence subgroups of  $SL_2(\mathbb{Z})$ , modularity and the Taniyama-Shimura conjecture, and Fermat's Last Theorem.

**Prerequisite(s):** MATH 5111 with a minimum grade of C- or MATH 5112 with a minimum grade of C-

**MATH 7362. Topics in Algebra. (4 Hours)**

Focuses on various advanced topics in algebra, the specific subject matter depending on the interests of the instructor and of the students. Topics may include homological algebra, commutative algebra, representation theory, or combinatorial aspects of commutative algebra. May be repeated without limit.

**MATH 7363. Topics in Algebraic Geometry. (4 Hours)**

Focuses on various advanced topics in algebraic geometry, the specific subject matter depending on the interests of the instructor and of the students. Topics may include integrable systems, cohomology theory of algebraic schemes, study of singularities, geometric invariant theory, and flag varieties and Schubert varieties. May be repeated without limit.

**MATH 7364. Topics in Representation Theory. (4 Hours)**

Offers topics in the representation theory of the classical groups, topics vary according to the interest of the instructor and students. Topics may include root systems, highest weight modules, Verma modules, Weyl character formula, Schur commutator lemma, Schur functors and symmetric functions, and Littlewood-Richardson rule. May be repeated up to five times.

**MATH 7371. Morse Theory. (4 Hours)**

Covers basic Morse theory for nondegenerate smooth functions, and applications to geodesics, Lie groups and symmetric spaces, Bott periodicity, Morse inequalities, and Witten deformation.

**Prerequisite(s):** (MATH 5122 with a minimum grade of C- or MATH 5122 with a minimum grade of D- ); MATH 7221 with a minimum grade of C- ; MATH 7301 with a minimum grade of C-

**MATH 7374. Riemannian Geometry and General Relativity. (4 Hours)**

Introduces Riemannian and pseudo-Riemannian geometry with applications to general relativity. Topics include Riemannian and pseudo-Riemannian metrics, connections, geodesics, curvature tensor, Ricci curvature and scalar curvature, Einstein's law of gravitation, the gravitational red shift, the Schwarzschild solution and black holes, and Einstein equations in the presence of matter and electromagnetic field.

**MATH 7375. Topics in Topology. (4 Hours)**

Offers various advanced topics in algebraic and geometric topology, the subject matter depending on the instructor and the students. Topics may include Morse theory, fiber bundles and characteristic classes, topology of complex hypersurfaces, knot theory and low-dimensional topology, K-theory, and rational homotopy theory. May be repeated without limit.

**Prerequisite(s):** MATH 5121 (may be taken concurrently) with a minimum grade of C-

**MATH 7381. Topics in Combinatorics. (4 Hours)**

Offers various advanced topics in combinatorics, the subject matter depending on the instructor and the students. May be repeated without limit.

**Prerequisite(s):** (MATH 5122 with a minimum grade of C- or MATH 5122 with a minimum grade of D- ); MATH 7222 with a minimum grade of C-

**MATH 7382. Topics in Probability. (4 Hours)**

Offers various advanced topics in probability and related areas. The specific subject matter depends on the interest of the instructor and students. May be repeated up to five times.

**MATH 7721. Readings in Topology. (4 Hours)**

Offers a reading course to be arranged between an individual student and instructor on a topic of their mutual choice. May be repeated without limit.

**MATH 7733. Readings in Graph Theory. (4 Hours)**

Offers a reading course to be arranged between an individual student and instructor on a topic of their mutual choice. May be repeated without limit.

**MATH 7734. Readings in Algebra. (4 Hours)**

Offers a reading course to be arranged between an individual student and instructor on a topic of their mutual choice. May be repeated without limit.

**MATH 7735. Readings in Algebraic Geometry. (4 Hours)**

Offers a reading course to be arranged between an individual student and instructor on a topic of their mutual choice. May be repeated without limit.

**MATH 7736. Readings in Discrete Geometry. (4 Hours)**

Offers a reading course to be arranged between an individual student and instructor on a topic of their mutual choice. May be repeated without limit.

**MATH 7741. Readings in Probability and Statistics. (4 Hours)**

Offers a reading course to be arranged between an individual student and instructor on a topic of their mutual choice. May be repeated without limit.

**MATH 7771. Readings in Geometry. (4 Hours)**

Offers topics in geometry that are beyond the ordinary undergraduate topics. Topics include the regular polytopes in dimensions greater than three, straight-edge and compass constructions in hyperbolic geometry, Penrose tilings, the geometry and algebra of the wallpaper, and three-dimensional Euclidean groups. May be repeated without limit.

**MATH 7962. Elective. (1-4 Hours)**

Offers elective credit for courses taken at other academic institutions. May be repeated without limit.

**MATH 8450. Research Seminar in Mathematics. (4 Hours)**

Introduces graduate students to current research in geometry, topology, mathematical physics, and in other areas of mathematics. Requires permission of instructor for undergraduate mathematics students. May be repeated without limit.

**MATH 8984. Research. (1-4 Hours)**

Offers an opportunity to conduct research under faculty supervision. May be repeated without limit.

**MATH 8986. Research. (0 Hours)**

Offers an opportunity to conduct full-time research under faculty supervision. May be repeated without limit.

**MATH 9000. PhD Candidacy Achieved. (0 Hours)**

Indicates successful completion of the doctoral comprehensive exam.

**MATH 9984. Research. (1-4 Hours)**

Offers an opportunity to conduct research under faculty supervision. May be repeated without limit.

**MATH 9990. Dissertation Term 1. (0 Hours)**

Offers dissertation supervision by members of the department.

**Prerequisite(s):** MATH 9000 with a minimum grade of S

**MATH 9991. Dissertation Term 2. (0 Hours)**

Offers dissertation supervision by members of the department.

**Prerequisite(s):** MATH 9990 with a minimum grade of S

**MATH 9996. Dissertation Continuation. (0 Hours)**

Offers dissertation supervision by members of the department.

**Prerequisite(s):** MATH 9991 with a minimum grade of S or Dissertation Check with a score of REQ